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INFERENCE OF LIGHTNING CHARGES BASED ON A MULTIPOLE EXPANSION MODEL

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1. INTRODUCTION

Multiple-station measurements of lightning-caused electric field changes (ΔEs) have been studied for years to determine the location, magnitude, and polarity of the charges deposited in a lightning flash (see for example, Workman and Holzer [1939]; Jacobson and Krider [1976]; Krehbiel and Brook [1979]; Maier and Krider [1986]; Koshak and Krider [1989]; Koshak and Krider [1993]). A variety of *inversion* techniques have been applied to derive charge solutions from the finite number of ground-based ΔE values. It is important to emphasis that these solutions are inherently nonunique since an infinity of charge distributions can produce identically the same set of ΔE observations [e.g., the fields outside *any* spherically symmetric sources of radius r and total charge Q, are identical]. Additionally, measurement errors can erode or otherwise distort the information contained in the ΔEs resulting in erroneous solutions. Depending on the inversion algorithm employed and the number and placement of measurements at the ground, numerical inversion of ΔEs can result in the amplification of measurement errors that lead to spurious solutions [Koshak and Krider, 1993].

In the present study, we review the primary means available for determining lightning charge solutions from ΔE observations and we introduce a new approach that is intimately related to the previous studies, but that offers some distinct advantages. The method, based on a formal multipole expansion of the lightning charge distribution, is tested using computer simulated sources and the geometry of the NASA Kennedy Space Center and USAF Eastern Space and Missile Center (KSC-ESMC) ground-based field mill network operated in 1978.

2. NONLINEAR ANALYSES

In nonlinear ΔE analyses, a charge model comprised of a certain number of charge parameters (i.e., the location and values of charges and/or charge moments) is used to describe the actual charges deposited in a flash. The optimum values of these parameters are determined so that the model fields at the ground agree favorably with the measured lightning field changes. Application of nonlinear model analyses have primarily been limited to simple point charge (Q) or point dipole (P) models. The mathematical form of the Q-model is:

$$\Delta E_i = \frac{2Qz}{4\pi\epsilon_o \left[(x-x_i)^2 + (y-y_i)^2 + z^2 \right]^{3/2}} \quad (1)$$

where ΔE_i is the model field change value produced at the i^{th} measurement site due to a model point charge Q located at (x, y, z). The i^{th} site is located at (x_i , y_i), and ϵ_0 is the permittivity of free space. Similarly, the fields produced at the ground due to a point dipole source (P-model) take the form:

$$\Delta E_i = \frac{1}{2\pi\epsilon_0 R_i^3} \left[P_z - \frac{3z(\mathbf{P} \cdot \mathbf{R}_i)}{R_i^2} \right] \quad (2)$$

where $\mathbf{R}_i$ = (x-x_i , y-y_i , z) points from the i^{th} field site to the model point dipole, $\mathbf{P} = (P_x, P_y, P_z)$. Hence, the Q- and P- models involve 4 (x, y, z, Q) and 6 (x, y, z, **P**) parameters, respectively.

In order to find the optimum model parameters, a nonlinear Chi-squared function of the form, $\chi^2 = \Sigma(\Delta E_i - g_i)^2/\sigma_i^2$ is minimized. Here, g_i and σ_i represent the *measured* field change and associated measurement error at the i^{th} measurement site, respectively. If the value of χ^2 is sufficiently small the model solution is assumed valid [Jacobson and Krider, 1976]. Generally speaking, the Q-model is used to describe ground discharges while the P-model is used to describe cloud discharges. In practice, both the Q- and P- models are used to analyze the same event, and the model that provides a better fit to the ΔE data (i.e. that produces a smaller value of χ^2) is chosen as the optimum solution.

3. LINEAR ANALYSES

In more recent years, a linear method has provided solutions in terms of general volume distributions of charge [Koshak, 1991; Koshak and Krider, 1993]. This approach has provided a framework for determining the information content in a set of ΔE measurements, and provides a general means for adding external constraints to the charge solution (i.e. constraints above and beyond those inherent in the measurements). A charge density distribution, $\Delta\rho(\mathbf{r})$, is related to fields at the ground, ΔE_i, in terms of a linear Fredholm integral equation of the first kind [Koshak and Krider, 1993]:

$$\Delta E_i = \int_V K_i(\mathbf{r})\ \Delta\rho(\mathbf{r})\ dV \qquad (3)$$

The kernel functions, $K_i(\mathbf{r})$, are geometrical functions derived from Coulomb's Law of electrostatics. For distant lightning, the kernel functions approach zero so that the field sensors no longer "see" the lightning-caused perturbations in charge density. For this reason it has been difficult to obtain accurate charge solutions for distant lightning sources [Koshak and Krider, 1991].

In this approach, there is no explicit search for the *location* of charges (i.e., of nonlinear parameters). Instead, the location of the unknown lightning source distribution is determined at a number of predefined points, $\mathbf{r} = \mathbf{r}_1, \ldots, \mathbf{r}_n$, above the measurement network. The volume of interest, V, is discretized with typically 2 km spatial resolution and the charges at each grid point in this 'solution grid' are taken as the only unknowns in the problem. The discrete form of the integral in (3) can be written as:

$$\mathbf{g} = \mathrm{K}\ \mathbf{f} + \boldsymbol{\sigma} \qquad (4)$$

where **f** is a column vector of j = 1,...,n unknown charges, **g** is a set of m measurements with errors, $\boldsymbol{\sigma}$, (i.e., $g_i = \Delta E_i + \sigma_i$, i = 1,...,m), and K is a (mxn) kernel matrix relating field to charge. This system is linear in the charges, **f**, and is amenable to various constrained inversion solutions [Twomey, 1977; Koshak and Krider, 1993].

4. BASIC LIMITATIONS

The nonlinear analyses described in section 2 have been used to study both ground and cloud flashes in a variety of storms centered near or over the KSC field mill network [Koshak and Krider, 1989; Koshak and Krider, 1991]. Because independent runs of both Q- and P- models must be used to analyze a single lightning event (and to discriminate a ground flash from a cloud flash) additional computing time is required.

In addition, many flashes are not fit by either a simple Q- or P- model. This can pose a problem for studies that depend on using ΔE analyses to estimate the time-averaged lightning currents and altitudes of charge centers in a thunderstorm [Driscoll et al., 1992]. To resolve a greater fraction of events, it is possible to use more complicated nonlinear models (e.g. models with 2, 3, or more charges) but these models are not unique, are difficult to constrain, and require more computing time.

Furthermore, determining the absolute minimum of the nonlinear χ^2 hypersurface is not always possible given conventional gradient-expansion minimization methods of the type described in Bevington [1976]. Physically unreasonable charge model parameters can result when a parameter search terminates in a *relative* minima of the χ^2 function.

By comparison, only one run of the linear method is normally required to analyze a lightning event. External constraints can be applied to suppress solution instability given the vast number of unknown charges inherent in this method. However, with only ground-based measurements, basic vertical dipole retrievals are, for instance, difficult [Koshak and Krider, 1993] and more work must be done to improve constraints, and computing time.

A computationally quick method that can be applied to a wide variety of lightning flashes is described in detail below. This approach can be viewed as a compromise between the simple nonlinear charge models and the general linear method. As such, the new approach has many virtues of the earlier methods, but is free of many of the limitations cited in the above paragraphs.

5. MULTIPOLE EXPANSION METHOD

It is well known that the potential outside an arbitrary distribution of charge can be expressed in terms of the moments of the charge distribution. Normally, one writes the potential outside the distribution as:

$$\phi(\mathbf{r}) = \int_{V'} \frac{1}{R} \rho(\mathbf{r}') \, dV' \qquad (5)$$

where $\mathbf{r}$ is an arbitrary point exterior to the charge distribution of volume V', $\mathbf{r}'$ is a variable of integration , and $R = |\mathbf{R}| = |\mathbf{r} - \mathbf{r}'|$ is the relative position vector. To write the potential in terms of the multipole moments of the charge distribution, $1/R$ is expanded in spherical harmonics [Jackson, 1975]. The volume projections of $\rho(\mathbf{r}')$ onto the different harmonic functions comprise the moments of the charge distribution.

For our purposes, it is instructive (and less tedious) to avoid the classic harmonic expansion of $1/R$, and deal directly with (3). Expanding $K_i(\mathbf{r})$ in a Taylor series about an arbitrary point $\mathbf{r}_o$ above the conducting plane, we immediately obtain the desired result:

$$\Delta E_i = \int_V [\, K_i(\mathbf{r}_o) + \partial_j K_i|_o \delta x_j + \frac{1}{2}\partial_j\partial_k K_i|_o \delta x_j \delta x_k + \ldots] \, \Delta\rho(\mathbf{r}) \, dV \qquad (6)$$

$$= K_i(\mathbf{r}_o)\, Q + \nabla K_i|_o \cdot \mathbf{P} + \ldots$$

where $Q = \int \Delta\rho(\mathbf{r}) dV$ (monopole moment), $\mathbf{P} = \int (\mathbf{r}-\mathbf{r}_o)\Delta\rho(\mathbf{r}) dV$ (dipole moment), and $j,k = 1, 2, 3$. The position vector, $\mathbf{r}$, has components, x_j , i.e., $(x_1 , x_2 , x_3) = (x, y, z) = \mathbf{r}$. Furthermore, $\partial_j = \partial/\partial x_j \Rightarrow (\partial/\partial x, \partial/\partial y, \partial/\partial z) = \nabla$, $\delta x_j = (x_j - x_{jo}) \Rightarrow (\mathbf{r}-\mathbf{r}_o)$. The expression (6) employs the Einstein convention for summing repeated indices. Note that the monopole moment is mapped onto the zeroth order derivative of $K_i(\mathbf{r})$, the dipole moment onto the first derivative of $K_i(\mathbf{r})$, the quadrupole onto the second derivative, and so on. For expansions of ϕ in this case, the interested reader is referred to Fitzgerald [1957].

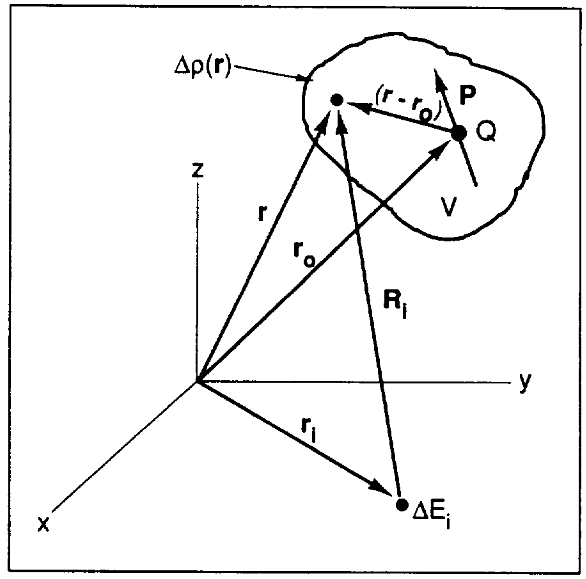

Figure 1. Geometry associated with a multipole expansion model.

The 'kernel coefficients' in front of each moment in (6) can be arranged into a vector, $\mathbf{a}_i = (K_i , \nabla K_i ,\ldots)$, that premultiplies a moment vector, $\boldsymbol{\mu} = (Q, \mathbf{P} ,\ldots)$, i.e., $\Delta E_i = \mathbf{a}_i \cdot \boldsymbol{\mu}$. Using vector notation for a set of $i = 1,\ldots,m$ field change measurements, $\mathbf{g} = \Delta\mathbf{E} + \boldsymbol{\sigma}$, we obtain:

$$\mathbf{g} = A\boldsymbol{\mu} + \boldsymbol{\sigma} \qquad (7)$$

where the $(m \times \infty)$ matrix $A = A(\mathbf{r}_o)$ has row vectors, $\mathbf{a}_i$.

Figure 1 summarizes the general problem implied by (7). Note from (4) the connection between the linear method and a general multipole expansion, i.e., $\Delta\mathbf{E} = \mathbf{g} - \boldsymbol{\sigma} = \mathbf{Kf} = A\boldsymbol{\mu}$. In effect, the matrix operator, $\mathbf{K}$, transforms the charge distribution, $\mathbf{f}$, into fields, whereas A transforms the *moments of* $\mathbf{f}$ into fields.

If the lightning charges occupy a sufficiently small volume, we may neglect the higher order terms in (7), and retain only the monopole and dipole terms. In other words, if $(\mathbf{r}-\mathbf{r}_o)$ is sufficiently small, the volume integral in (3) can be accurately computed using a linear approximation of $K_i(\mathbf{r})$ throughout V. With this truncation, $\boldsymbol{\mu}$ becomes a 4-vector with components (Q, **P**), and the net field change at the ground, $\Delta E_i = (A\boldsymbol{\mu})_i$, is identically the sum of the changes given by (1) and (2). Hence, this truncated 'μ-model' or 'QP-model' involves 7 parameters, $(\mathbf{r}_o,Q,\mathbf{P})$, with the point charge, Q, and point dipole, **P**, co-located at the point, $\mathbf{r}_o$.

Now, instead of searching for all of the optimum values of $(\mathbf{r}_o,Q,\mathbf{P})$ that minimize a χ^2 function (see section 2) we note that the optimum moments at any location, $\mathbf{r}_o$, are given by direct inversion of (7), i.e., $\boldsymbol{\mu}(\mathbf{r}_o) = (A^tA)^{-1}A^t(\mathbf{g}-\boldsymbol{\sigma})$. Here, A^t is taken as the 'transpose of A'. If $\mathbf{r}_o$ closely approximates the true source location, a good fit to the data will be achieved and the value of the error function, $e(\mathbf{r}_o) = [\mathbf{g} - A(\mathbf{r}_o)\boldsymbol{\mu}(\mathbf{r}_o)]^2$, will be small. Since A^tA is only a (4 x 4) matrix, it is practical to scan a large volume over the measurement network, and determine the value of $\mathbf{r}_o$ that makes $e(\mathbf{r}_o)$ a minimum.

Hence, this procedure avoids the problem of relative minima since a thorough grid search is used to estimate the optimum source location. The optimum source moments at each location are obtained from numerical matrix inversion. The speed of scanning can be improved if all inverse matrices are computed in advance, stored on disk, and then read directly into memory during a grid search.

The method we have described also allows one to retain moments beyond the dipole term if desired. For instance, the traceless and symmetric quadrupole moment tensor would only involve 5 additional moments. This implies a (9x9) matrix inversion, $(A^tA)^{-1}$, at each point, $\mathbf{r}_o$.

It has been found that for several tens of kilometers outside the perimeter of the measurement network, the elements of A become small, and the matrix A^tA becomes ill-conditioned. In this case, it will take more computer time to compute the inverse of A^tA, and the moment solution may be in error due to excessive magnification of measurement errors. A constrained least-squares approach that stabilizes solutions for distant lightning can be written as:

$$\boldsymbol{\mu}(\mathbf{r}_o) = (A^tA+\gamma H)^{-1}A^t(\mathbf{g} - \boldsymbol{\sigma}) \qquad (8)$$

Here, H is a constraint matrix and γ is a factor that determines how strongly the external constraints in H are weighted (see Twomey [1977] and Koshak and Krider [1993] for more details on constrained linear inversion techniques and error magnification).

In this paper, we only consider lightning flashes that are over the network or within about 10 kilometers of the network perimeter. The optimum values of γ and the form of the constraint matrix optimum for distant lightning will not be considered in detail. Since a measurement network extracts little information about lightning sources several tens of kilometers away, external constraints (not measurements) become the driving factor in determining the character of a solution. Nonetheless, we will choose H = I (identity matrix) in order to impose the constraint that the sum of the squares of the moments, $\boldsymbol{\mu}^2$, is not exceedingly large. With this constraint, a value $\gamma = 10^{-9}$ has been found to produce optimum results in our domain of interest.

6. SIMULATED TESTS OF THE METHOD

In this section, we test the expansion method's ability to retrieve known, computer simulated lightning sources. To do this, we consider the ground-based field mill network at the NASA Kennedy Space Center and USAF Eastern Space and Missile Center (KSC-ESMC) operated in 1978 and shown below in Figure 2.

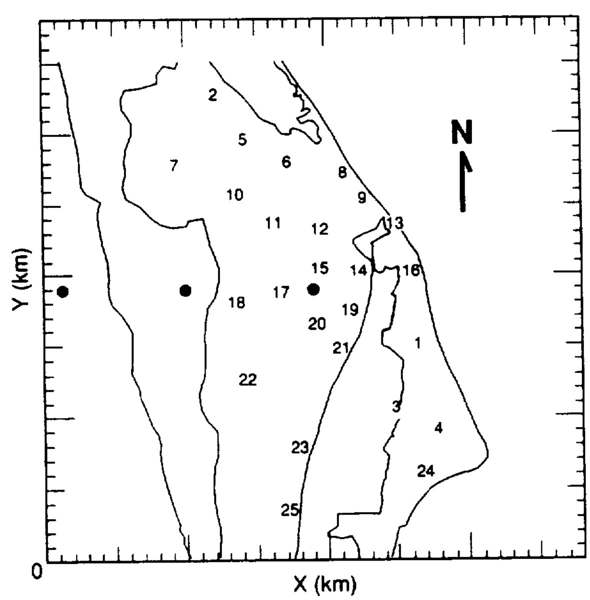

Figure 2. The KSC-ESMC field mill network operated in 1978. Solid dots indicate (x,y) locations of simulated lightning sources.

Since it is not uncommon that a few mills are inoperative during a storm, we have removed mills 5, 12, 15, and 17 from our simulated analyses. These 4 mills did not operate properly during a small storm occurring over the network on July 11, 1978 [Koshak, 1991].

The 3 solid dots that are oriented east to west in Figure 2 indicate the placement of known sources relative to the field mill network. At these locations, the source altitudes were z = 3, 5, 7,...,17 km. A total of 100 known sources were generated at each of the 24 locations, i.e., a total of 2400 sources were analyzed. Each source was assigned a monopole moment and a dipole moment. The monopole was varied randomly between 0 and 50 C and each cartesian component of the dipole moment vector was varied between 0 and 500 C km.

To find the minimum value of e, we scanned a large volume above the KSC-ESMC network with 2 km resolution (i.e., x and y each ranged between: 2, 4, ..., 36 km; and z = 2, 4,..., 18 km). From this scan we obtained a minimum value, $e(\mathbf{r}_{min})$. Next, we continued with a higher resolution (500 meter) scan centered ±1.5 km about $\mathbf{r}_{min}$ in each of the three cartesian directions. Using a SGI XS-4000 workstation, a single lightning source took less than 11 seconds to analyze. This is noteworthy because we did *not* read inverse matrix elements from previously generated data files (see section 5).

Tables 1 and 2 summarize the statistics of the errors in the retrieved QP-solutions. In Table 1, no error was added to the simulated field changes, and in Table 2 a 5% random error was added to the ΔEs. The location and dipole errors are determined by taking the magnitude of the difference vector, i.e., $|\mathbf{r}_{ret} - \mathbf{r}_{kno}|$, and $|\mathbf{P}_{ret} - \mathbf{P}_{kno}|$, where the subscripts 'ret' and 'kno' denote retrieved and known quantities, respectively. The monopole error is simply the absolute value of the scalar difference between the retrieved and known monopole values. Median distance errors with asterisks indicate that a majority of the 100 sources at that location had no location error.

X,Y	Z	Location Error (km)			Monopole Error (C)			Dipole Error (Ckm)		
(km)	(km)	Median	Mean	Std Dev.	Median	Mean	Std Dev.	Median	Mean	Std Dev.
	17	0.25	0.26	0.26	0.34	3.94	4.84	14.59	48.13	52.68
	15	0.00	0.50	1.52	0.04	2.55	8.39	0.30	44.00	134.04
	13	*0.00	0.04	0.42	0.00	0.02	0.18	0.01	0.86	8.53
(1,19)	11	0.00	0.31	0.40	0.00	12.68	31.69	0.01	67.53	122.52
	9	*0.00	1.19	2.59	0.13	7.57	16.26	0.64	95.14	199.86
	7	0.00	0.08	0.35	0.00	0.99	5.67	0.00	14.12	63.42
	5	0.00	0.40	1.17	0.00	14.58	28.24	0.01	70.86	142.75
	3	1.58	2.21	2.72	9.28	25.06	29.71	171.65	239.54	274.54
	17	0.00	0.71	2.01	0.03	0.94	2.55	0.33	34.09	89.11
	15	*0.00	0.06	0.57	0.00	0.17	1.68	0.01	5.73	57.24
	13	0.00	0.29	0.67	0.00	1.66	3.58	0.01	30.37	59.31
(10,19)	11	0.00	0.48	1.40	0.03	3.80	25.73	0.25	42.70	127.25
	9	*0.00	0.05	0.52	0.00	0.10	0.93	0.00	3.38	33.74
	7	0.00	0.30	0.78	0.00	3.04	8.64	0.01	43.08	105.51
	5	0.00	0.80	2.22	0.03	2.03	5.74	0.26	60.69	169.41
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00
	17	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.01
	15	0.00	0.44	1.65	0.01	0.85	1.58	0.05	32.43	93.38
	13	0.00	0.37	1.71	0.03	0.31	1.26	0.41	18.77	82.15
(19,19)	11	*0.00	0.13	0.96	0.00	0.07	0.50	0.01	6.11	43.22
	9	0.00	0.20	0.28	0.00	0.80	1.60	0.03	21.33	33.60
	7	0.00	0.50	1.81	0.03	0.77	2.48	0.40	30.38	108.74
	5	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00
	3	0.00	0.53	1.68	0.00	1.20	3.84	0.02	33.84	79.20

Table 1. Statistics of 2400 simulated lightning sources having a monopole and dipole moment. The median, mean, and standard deviation are computed from 100 randomized sources at each of the 24 (x,y,z) source locations. No error was added to the simulated ΔEs.

X,Y (km)	Z (km)	Location Error (km) Median	Location Error (km) Mean	Location Error (km) Std Dev.	Monopole Error (C) Median	Monopole Error (C) Mean	Monopole Error (C) Std Dev.	Dipole Error (Ckm) Median	Dipole Error (Ckm) Mean	Dipole Error (Ckm) Std Dev.
	17	0.50	0.39	0.35	3.78	5.79	6.40	60.24	67.61	65.81
	15	1.12	2.18	2.59	12.77	37.76	139.45	135.65	262.61	413.02
	13	0.00	0.01	0.07	0.72	1.14	1.17	10.54	14.25	13.83
(1,19)	11	0.50	0.51	0.92	4.83	14.66	25.52	44.01	101.24	152.66
	9	1.12	2.27	2.55	25.49	44.09	95.23	190.75	270.93	295.75
	7	0.00	0.18	0.63	1.14	3.75	9.81	13.75	45.72	104.69
	5	0.50	0.76	1.31	13.49	36.82	56.46	90.70	159.62	201.99
	3	2.24	3.72	3.32	41.34	60.15	61.38	348.00	402.93	263.14
	17	1.50	2.72	3.18	6.67	8.11	6.91	130.05	180.80	161.52
	15	0.00	0.24	0.32	0.56	1.47	1.96	13.65	35.58	43.16
	13	0.50	0.61	0.74	2.35	4.70	6.86	50.20	75.06	90.19
(10,19)	11	1.12	1.99	2.38	6.35	9.37	10.35	120.40	168.67	169.43
	9	0.00	0.14	0.55	0.62	1.26	1.78	8.69	23.09	43.30
	7	0.50	0.45	0.41	3.10	5.01	7.12	57.54	69.91	75.22
	5	0.87	2.40	3.05	7.74	14.61	24.49	104.35	207.42	230.75
	3	0.00	0.05	0.21	0.73	1.14	1.66	8.68	16.62	26.37
	17	0.71	0.84	1.07	1.51	2.36	2.67	57.07	72.63	84.89
	15	0.71	1.17	1.58	2.65	4.26	5.01	61.91	109.59	139.59
	13	1.54	2.36	2.69	4.63	5.99	5.23	98.26	165.43	180.69
(19,19)	11	0.50	0.64	1.15	1.10	2.04	2.57	37.29	58.61	81.70
	9	0.71	1.02	1.44	2.19	4.39	5.25	50.83	99.05	117.58
	7	1.23	2.68	3.41	4.49	7.02	9.17	105.05	177.84	187.63
	5	0.00	0.33	0.40	1.09	1.66	1.77	21.47	41.89	46.59
	3	0.71	1.03	1.88	2.03	5.80	23.22	55.14	89.67	148.76

Table 2. Same analyses as given in Table 1 except that a 5% random error was added to the simulated ΔEs.

7. DISCUSSION

A few sources (of the 100 randomized at each location) resulted in poor retrievals and large solution errors. This in turn has biased the *mean* values in Tables 1 and 2 toward the high end. A more representative statistic is the *median* error. Since the majority of sources at a point were retrieved accurately, the median error values are almost always smaller than the mean errors, and in most cases are substantially smaller.

Although the addition of a 5% error clearly increases solution errors (i.e., compare errors in Table 1 to those in Table 2), the standard deviation of the errors given in Table 1 indicate that there are occasionally poor retrievals even when exact ΔE values are used. The ability of the network to resolve the values of Q and **P** at a given point depend on the particular values chosen for Q and **P** and the location of the source. This is consistent with the fact that ΔE inversions produce solutions that are inherently nonunique, i.e., it is possible to find more than one QP-solution for a given QP-source. Note that the effects of source location are particularly pronounced for the case of the distant, low-altitude sources [see Tables 1 and 2 for sources located at **r** = (1 km, 19 km, 3 km)].

Overall, the multipole expansion method produces favorable retrievals. When error is added to the ΔEs (Table 2), median distance errors are all within about 2 km (except for the distant, low-altitude sources), and errors are generally smaller over the network. Monopole errors for sources over the network are within 5 C, and errors in a single component of **P** are within about 105 Ckm/$3^{1/2} \simeq$ 60.6 Ckm.

8. SUMMARY

Presently, there is a need for a real-time

Lightning Charge Mapper (LCM) system at the NASA Kennedy Space Center (KSC) to improve warnings against atmospheric electrical hazards to space vehicle operations. An LCM system would, in general terms, require ground-based observations of ΔE, and a quick and accurate computer algorithm to analyze the ΔEs to determine the lightning charge locations.

In this paper, we have described a quick yet thorough method for determining the charge moments and location of a lightning flash. Given the deficiencies in conventional linear and nonlinear ΔE analyses (section 4), we believe that this new approach is best suited for meeting the requirements of an LCM.

The power of this method resides in the fact that the optimum charge moments do not have to be *searched* for (in the sense of χ^2 minimization procedure), but are instead computed directly from *linear inversion*. No matter how many moments one attempts to find, the expansion method only requires searching for the 3 (nonlinear) spatial parameters that describe lightning location. Although we could have employed gradient/expansion methods to speed up this search, our direct solution for charge moments are sufficiently quick to allow for a high resolution grid search of the optimum spatial parameters. Because of this, the problems of relative minima are avoided.

Finally, we have tested the multipole expansion method using computer simulated lightning sources and have found reasonably small errors in the derived monopole, dipole, and lightning location when the source is placed over the network. In the future, we will apply this method to actual storm data [e.g. July, 11 1978, and storms from the Convection and Precipitation/Electrification Experiment (CAPE)] and we will compare our results with other ΔE analyses.

9. ACKNOWLEDGEMENTS

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